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AN IMPROVED INCREMENT-CORE METHOD FOR PREDICTING GROWTH OF FOREST STANDS

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INTRODUCTION

Foresters, responsible for management of forest tracts or for appraising changes in timber supplies, must cope with the problem of volume-growth estimation. They cannot always base such estimates on periodic remeasurements of stands. Commonly they estimate growth from increment-core measurements, but some of the methods in use are only rough approximations and others are too complex and time-consuming.

Proposed is a method of estimating growth which has proved to be reliable and yet requires a minimum of office and field work. To use it, only a rudimentary knowledge of statistical procedures is necessary. The method can be applied to both large and small areas and, with some modifications, can be used to obtain results directly in the field. With it comparisons of local or species differences are easily made.

GENERAL PROCEDURES AND MEASUREMENTS REQUIRED

Briefly stated the proposed method uses the principle of stand projection but without the intricate bookkeeping adjustments often associated with stand projection.

The application of the method requires first, the preparation of stock tables showing volumes by tree diameter classes. On large surveys such tables are usually prepared for each separate condition, such as type, size class of stand, and possibly each density class. It is also necessary to know the number of trees in the diameters just below merchantable size in order to estimate ingrowth on trees expected to become merchantable in the given period of time.

1/ Maintained at University Farm, St. Paul, Minnesota, in cooperation with the University of Minnesota.

2/ Revision of paper presented at Forest Survey Second Techniques Meeting, Eagle River, Wisconsin, September 29 to October 10, 1947.

3/ Silviculturist and Statistician, respectively.

The second requirement is to obtain borings to determine diameter increment and total age of sample trees of various sizes taken at random in the area. These will be used to estimate the probable future increment in diameter for trees of each species and diameter class. The number of borings needed will depend on the accuracy desired and the number of condition classes for which growth predictions are required. (See appendix for indication of number of borings needed.) Sample trees will be used also to estimate average heights corresponding to various diameters.

The third step is to convert this average diameter growth into growth percent in volume. This requires making allowance for height growth and for the fact that growth in volume is more rapid than growth in diameter.

Next, these growth percentages must be applied to the stock tables compiled from sample plots, thus obtaining gross increment in volume.

Finally, net increment is obtained by making deductions for estimated mortality and cull losses. Records for these deductions are obtained in the field for trees of various species and diameters over some definite period of time, usually one to ten years.

DETERMINATION OF DIAMETER AND HEIGHT INCREMENT

Analysis of the data obtained from 2,200 red oak sample trees in southwestern Wisconsin indicated a strong correlation of the current diameter increment with the average 10-year growth during the life of the tree and with total age. The average 10-year growth obviously reflects the combined effect of the various growth factors, such as site, crown class, stand density, etc. It is high if these factors were favorable, and low if they were unfavorable. Since these factors continue to affect the tree to a large degree, a close relationship exists between the average 10-year growth and both current and future growth. A tree which has grown slowly in the past, barring release, will probably continue to grow slowly during the next 10 years. In even-aged stands of intolerant species, diameter increment generally decreases with age, and growth for the past 10 years will, as a rule, be smaller than for the average 10-year period.

Although a high correlation exists between the past 10 years' growth and that for the average 10-year period (henceforth called "decadal growth" in this paper), a scatter diagram generally shows considerable variation of individual trees from the trend line. An important measurable factor causing this variation is age. If a curve based on these deviations from the trend line is plotted over age, it immediately becomes apparent that growth estimates based on decadal growth only must be increased for the younger trees and reduced for the older trees.

When these two curves have been drawn for a given species, a table can be prepared showing 10-year diameter increment for each d.b.h. and age group (table 1). A detailed description of the process leading to the preparation of this table is shown in the appendix.

The chief advantage of this method is that the decadal growth concept automatically allows for the effect of such factors as site, age, density, etc., without requiring the intricate harmonization of numerous curves which otherwise would have to be employed.

Table 1.--Future 10-year diameter increment by age and diameter class, for fully stocked red oak stands, Wisconsin

Present age	Future 10-year diameter increment when present diameter breast height is -													
	2	4	6	8	10	12	14	16	18	20	22	24	26	28
Years	Inches													
15	1.68	3.56												
20	.99	2.15	3.31											
25	.67	1.47	2.27	3.07										
30	.45	1.02	1.59	2.16	2.73									
35	.36	.81	1.26	1.71	2.17									
40	.30	.68	1.06	1.43	1.81	2.19								
45		.59	.92	1.25	1.58	1.91	2.24							
50		.51	.81	1.11	1.40	1.70	2.00	2.30						
55		.47	.74	1.01	1.28	1.55	1.81	2.08	2.35					
60		.42	.67	.92	1.17	1.42	1.67	1.91	2.16	2.40				
65		.38	.61	.84	1.07	1.30	1.54	1.77	2.00	2.23	2.46			
70			.57	.78	1.00	1.21	1.42	1.63	1.84	2.06	2.27	2.49		
80			.48	.67	.86	1.05	1.24	1.43	1.62	1.81	2.00	2.19	2.38	2.56
90			.43	.60	.77	.93	1.10	1.27	1.44	1.60	1.77	1.94	2.11	2.27
100				.53	.68	.83	.98	1.14	1.29	1.44	1.59	1.74	1.90	2.05
110				.47	.61	.75	.89	1.03	1.17	1.31	1.45	1.59	1.73	1.87
120				.43	.55	.68	.81	.94	1.06	1.19	1.32	1.44	1.57	1.70
130					.50	.62	.74	.86	.98	1.10	1.22	1.34	1.46	1.58
140					.47	.58	.69	.80	.91	1.02	1.13	1.24	1.35	1.46

In a cover type composed of several species, the proposed method of determining diameter increment should be applied to each species for which sufficient data are available. One or several can be considered key species, to which the growth of minor species can then be compared. This can be done by plotting the recorded last 10 years' increment of a minor species over the increment for the key species for the same age and diameter. The resulting curve would indicate the correction factor to be applied. If the points line up below the 45-degree trend, then a reduction must be made for slower growth. If the points are above the 45-degree trend, allowance must be made for faster growth. If the points scatter below and above the line, the minor species can be considered as growing at the same rate as the key species.

Any cutting or release adds more variability to the trend line of the last 10 years' diameter increment plotted over decadal growth. If such cuttings are not concentrated or do not extend over large territories, their effect is absorbed in the record of the increment cores obtained. The computed increment for the next 10 years will apply on the assumption that the same rate of cutting is maintained. If, however, the cuttings are restricted and occupy certain known blocks in the area, their effect on these blocks should be determined by means of special studies. The proposed method of plotting the diameter increment over decadal growth should be applied to the areas affected separately from the undisturbed stands.

The determination of volume growth also requires the estimation of height increment. It is seldom practical, however, to obtain height increments by direct measurement in the field and indirect methods are therefore required.

In uneven-aged stands, where both the diameters and heights are correlated with age, it is generally assumed that the height-over diameter curve remains unchanged over a long period of time. Therefore, under these conditions the expected increment in height for the period may be roughly estimated from a height curve derived from sample tree data. The height increment for a given d.b.h. is estimated as the difference between the present height and that corresponding to the expected diameter at the end of the period.

When the height-over-diameter curves change considerably with time, as is the case in even-aged stands, height increment would be underestimated by this method, particularly for the younger stands. Moreover, if the curve is based entirely on one age class, estimates for that age class would be especially low.

A better method of estimating height growth would require the construction of a series of height-over-diameter curves, one for each age class. If such curves are made, the height increment can then be estimated as the difference between the readings from present and future age curves.

This procedure would require a large number of observations. Since such a series of age curves will seldom be made, experience tables have been devised for most Lake States species showing expected increments in both total and merchantable heights. They are based on a large number of observations obtained in conjunction with stem analyses, taper measurements, and growth and yield studies.

Although the increments presented in the tables below are rather generalized, they are considerably more accurate than a single height curve or a series of curves built up from small samples.

Table 2.--Expected 10-year increment in total height for various species groups and for specified 10-year diameter increments, by age 1/

(For Growth in Cubic Feet)

Species group	10-year diameter increment	10-year total height increment at ages-									
		20	30	40	50	60	80	100	120	140	160
	Inches	Feet									
Slow-growing group (Black spruce, red oak)	0.5	6	5	4	3	3	2	2	2	1	1
	1.0	8	7	6	5	4	3	2	2	1	1
	2.0	10	9	7	6	6	4	3	3	2	2
	3.0	13	10	9	8	7	5	4	3	3	2
General group (Northern hardwood, aspen, jack pine, red pine, balsam fir)	0.5	9	8	6	5	4	4	3	3	2	2
	1.0	11	9	8	7	6	5	4	4	3	3
	2.0	14	12	10	8	7	6	4	5	4	4
	3.0	16	14	12	11	9	7	6	5	4	4
Fast-growing group (white pine)	0.5	14	13	11	9	7	5	4	3	3	2
	1.0	16	14	12	11	9	6	5	3	3	2
	2.0	17	16	14	12	13	8	6	5	4	3
	3.0	19	17	16	14	13	10	7	6	5	4

1/ Applicable primarily to average sites. Increase 20 percent for good sites; reduce by the same amount for poor sites.

Table 3.--Expected increment in merchantable height for specified diameter increments, by d.b.h. class and number of 16-foot logs 1/

(For Growth in Board Feet)

D.b.h.	Diameter increment	Merchantable height increment according to - Number of 16-foot logs 2/						
		1/2	1	1-1/2	2	3	4	5
	Inches	Feet						
10	.5	1	3	5	7			
	1.0	4	8	12	16			
	2.0	8	14	18	22			
	4.0	14	24	30	35			
12	.5	1	2	2	3	4		
	1.0	2	4	5	6	8		
	2.0	3	7	9	11	14		
	4.0	5	10	13	16	21		
14	.5	0	0	1	1	1	2	
	1.0	0	2	2	2	3	4	
	2.0	1	4	5	6	6	7	
	4.0	2	7	8	9	10	12	
16	.5	0	0	0	1	1	1	3
	1.0	0	1	1	1	2	3	5
	2.0	0	2	2	3	3	4	7
	4.0	1	3	3	4	5	7	10
20	.5	0	0	0	0	0	1	2
	1.0	0	0	0	0	1	2	3
	2.0	0	0	0	0	2	3	4
	4.0	0	0	0	1	3	4	6
30	.5	0	0	0	0	0	1	1
	1.0	0	0	0	0	0	1	2
	2.0	0	0	0	0	1	2	3
	4.0	0	0	0	1	2	3	4

1/ The use of this table is not restricted to any specific period.
2/ Length to usable top, not less than 8.0 inches inside bark.

Table 4.--Expected increment in merchantable height for specified diameter increments, by d.b.h. class and number of 8-foot bolts 1/

(For Growth in Cords)

D.b.h.	Diameter increment	Merchantable height increment according to - Number of 8-foot bolts 2/							
		1	2	3	4	5	6	7	8
Inches	Inches				Feet				
6	.5	1	3	4					
	1.0	3	7	9					
	2.0	6	13	16					
	4.0	10	24	29					
8	.5	0	1	2	3	4			
	1.0	1	2	4	6	7			
	2.0	1	3	6	10	12			
	4.0	2	7	13	19	23			
10	.5	0	0	0	1	2	2	3	
	1.0	0	0	1	3	5	5	6	
	2.0	0	2	4	7	9	10	10	
	4.0	1	3	6	10	14	15	16	
12	.5	0	0	0	0	1	2	2	2
	1.0	0	0	0	1	2	3	4	4
	2.0	0	0	0	2	3	6	6	7
	4.0	0	1	2	5	8	10	11	12
16	.5	0	0	0	0	0	0	1	2
	1.0	0	0	0	0	0	1	2	2
	2.0	0	0	0	0	1	2	4	4
	4.0	0	0	0	1	2	4	6	7
20	.5	0	0	0	0	0	0	0	0
	1.0	0	0	0	0	0	0	0	0
	2.0	0	0	0	0	0	1	2	2
	4.0	0	0	0	0	0	1	2	3

1/ The use of this table is not restricted to any specified period.
2/ Length to a usable top, not less than 4.0 inches inside bark.

CONVERSION OF DIAMETER INCREMENT INTO VOLUME GROWTH

After diameter and height increment have been determined, they must be converted into volume growth by a series of steps, the first of which involves calculation of volume-growth percent.

Calculation of Volume-Growth Percent

The current growth percent of any tree in terms of volume, when no change in form is assumed, can be computed from the general formula:

$P = a\frac{d}{D} + b\frac{h}{H}$ where

P = growth percent

d = the expected increment for the period in d.b.h. (inches)

D = the present d.b.h. (inches)

h = the expected increment for the period in merchantable height (feet)

H = the present merchantable height (feet)

a and b = factors converting diameter and height increment into tree volume increment.

These a and b factors vary for the different d.b.h. classes and for different diameter increments (table 5).

To illustrate the application of these growth factors in determining volume-growth percent for the International rule, the following example may be used:

Present d.b.h. (D) = 20 inches

Diameter increment (d) = 4 inches

Present merchantable height (H) = 48 feet

Height increment (h) = 3 feet

$P = a\frac{d}{D} + b\frac{h}{H} = 230\frac{4}{20} + 111\frac{3}{48} = 52.9$ percent

Table 5.--Growth factors for determining volume-growth percent

FOR CORDWOOD VOLUME

Diameter breast height	Growth factors when diameter growth in inches is -													
	0.5		1.0		2.0		3.0		4.0		5.0		6.0	
	a	b	a	b	a	b	a	b	a	b	a	b	a	b
Inches														
6	172	120	182	124	198	136	210	156	218	182	226	215	232	254
8	169	115	177	120	190	132	199	148	205	170	210	194	213	219
10	167	110	174	114	184	126	190	141	195	159	198	179	200	199
12	166	102	170	106	177	116	182	130	185	144	188	158	190	174
14+	164	102	166	106	171	116	175	130	178	144	182	158	184	174
FOR BOARD-FOOT VOLUME, SCRIBNER RULE														
10	300	92	300	100	300	119	304	143	312	170	326	198	350	228
12	281	90	281	96	281	112	281	132	287	154	298	177	311	201
14	268	87	268	92	268	106	270	122	276	139	285	156	298	174
16	259	84	259	90	259	101	259	114	265	127	272	141	283	155
18	248	83	248	88	248	98	249	110	253	121	262	133	273	146
20	238	82	238	86	238	95	239	105	244	115	252	126	262	137
22	236	81	236	84	236	92	237	101	241	111	249	121	260	131
24	234	80	234	83	234	90	234	99	238	108	246	116	255	126
26	232	79	232	82	232	89	232	96	236	104	242	112	250	120
28	230	79	230	81	230	87	230	93	233	100	238	107	245	114
30	228	78	228	80	228	85	228	91	229	97	233	103	239	110
40+	219	77	219	78	219	82	219	87	222	92	226	98	230	104
FOR BOARD-FOOT VOLUME, INTERNATIONAL RULE														
10	232	85	232	92	236	109	244	130	256	151	269	174	282	195
12	232	84	232	89	233	103	241	119	252	137	263	154	274	173
14	230	82	230	88	231	101	237	115	245	130	255	145	265	161
16	227	81	227	87	229	99	234	112	241	126	249	140	259	155
18	224	81	225	85	227	96	231	107	236	119	243	131	250	143
20	221	80	221	84	224	93	226	101	230	111	235	120	241	130
22	219	79	219	83	222	91	224	99	229	108	234	117	240	126
24	217	79	217	82	220	89	223	97	228	105	233	113	239	122
26	215	78	215	81	218	88	222	95	227	102	232	110	238	118
28	213	78	213	80	216	86	221	93	226	99	231	106	237	113
30	211	78	212	80	215	85	220	91	225	96	230	102	236	109
40+	205	77	206	78	209	81	213	85	218	90	224	95	229	100

Correction for Form Class

The a and b factors given above apply specifically to Lake States conditions, where the average run of merchantable timber lies between 10- and 40-inch d.b.h. and one to four 16-foot logs in merchantable trees and has form classes of 75 to 80 (table 6).

Table 6.--Average form classes[1/] for the Lake States timber

Diameter breast height	Form class when merchantable height in number of 16-foot logs is -			
	1	2	3	4
Inches				
10	80			
12	79	79		
14	78	78		
16	78	78	80	
18	78	78	79	80
20	78	77	78	79
22	78	77	78	79
24	78	77	77	79
26	78	77	77	78
30	78	76	76	78
34	78	76	76	77
40	78	75	75	76

1/ Form class is the percentage ratio between the diameter, inside bark, at the top of the first 16-foot log and the diameter, outside bark, at breast height.

When the pattern of form-class distribution by tree sizes differs from that shown, correction factors should be applied as follows:

$$P_c = c\,(100 + P) - 100$$

The correction factor (c) has been computed for almost any difference or changes in form class (table 7).

Table 7.--Correction factors to be applied when form classes differ from those used in the Lake States

Difference in present form class 1/	Correction (c) factors when difference in future form class is: 1/										
	-5	-4	-3	-2	-1	0	1	2	3	4	5
-5	1.000	1.035	1.071	1.105	1.141	1.176	1.212	1.247	1.282	1.318	1.353
-4	.966	1.000	1.034	1.068	1.102	1.136	1.170	1.205	1.239	1.273	1.307
-3	.934	.967	1.000	1.033	1.066	1.099	1.132	1.165	1.198	1.231	1.264
-2	.904	.936	.967	1.000	1.032	1.064	1.096	1.128	1.160	1.191	1.223
-1	.876	.907	.938	.969	1.000	1.031	1.062	1.093	1.124	1.155	1.186
0	.850	.880	.910	.940	.970	1.000	1.030	1.060	1.090	1.120	1.150
1	.825	.854	.883	.913	.942	.971	1.000	1.029	1.058	1.087	1.117
2	.802	.830	.858	.887	.915	.943	.972	1.000	1.028	1.057	1.085
3	.780	.807	.835	.862	.890	.917	.945	.972	1.000	1.028	1.055
4	.759	.786	.813	.839	.866	.893	.920	.946	.973	1.000	1.027
5	.739	.765	.791	.817	.843	.870	.896	.922	.948	.974	1.000

1/ New form class minus form class given in table 6.

In the example previously given for determining volume-growth percent, the Lake States form class for the present was 78 and for the future 77. If the form class at present were 80 rather than 78, and it were assumed to increase to 82 in the future, then, according to the above table, the present difference would be 80 minus 78 or 2, and the future difference would be 82 minus 77 or 5. Accordingly, the c factor would be 1.085. Applying the formula given above:

$$P_c = 1.085\ (100 + 52.9) - 100 = 65.9$$

With the correction for form class an accurate growth percent can be obtained for any local condition or for individual trees. The formula, therefore, could be applied in other regions as well as the Lake States. Furthermore, the tables of factors for converting diameter and height increment into volume growth developed for this method will be useful to all who wish to determine forest growth, regardless of the prediction method used.

Application to Uneven-Aged Stands

The method can be applied to any breakdown by the survey condition classes. Whether the breakdown is by size class, density, type, etc., or a combination of these factors, a knowledge of stand age is required by diameter class. The application is simpler, therefore, to even-aged stands. Under conditions where the age varies with d.b.h., a curve should be constructed from sample-tree data showing average age of each d.b.h. group in each condition class. This would make possible the application of a table of future 10-year diameter increment by age and d.b.h. class (table 1) to uneven-aged stands.

Calculation of Gross Growth

Gross growth includes the growth of trees now merchantable plus the volume of smaller trees that may be expected to advance into merchantable size classes during the period of the estimate.

Growth of Merchantable Trees

After the stand and stock table has been made, the growth percent is applied to each diameter class and the total growth estimated (table 8).

Table 8.--Calculation of gross growth in board feet (International 1/4-inch rule) for a 60-year-old, fully stocked red oak stand, medium site

D.b.h. class	Trees per acre	Gross volume per acre	Average 10-year diameter increment	Components of growth percent formula 1/				Growth percent	Gross volume growth per acre
				$\frac{d}{D}$	a factor	$\frac{h}{H}$	b factor		
Inches	Number	Bd. ft.	Inches					Percent	Bd. ft.
12	34	2,312	1.42	.118	232	.280	95	54.0	1,248
14	11	1,276	1.67	.119	231	.188	97	45.7	583
16	3	534	1.91	.119	229	.105	98	37.5	200
18	1	240	2.16	.120	228	.048	98	32.1	77
Total	49	4,362	....	....	...	....	..	....	2,108

1/ d = diameter increment, D = present d.b.h., h = height increment, H = present merchantable height. a and b = factors for converting diameter and height increment into tree volume increment (see table 5).

Ingrowth

Ingrowth may be calculated from a table based upon increment cores, and showing what percentage of the total number of trees in any d.b.h. class may be expected to advance into larger classes during a given period (table 9).

Table 9.--Probability of trees in any 2-inch d.b.h. class growing into larger d.b.h. classes, in relation to their average increment in diameter 1/

Average increment in d.b.h.	Portion of total number of trees that will --				
	Remain in same d.b.h. class	Grow into indicated larger 2-inch d.b.h. class			
		Next	Second	Third	Fourth
Inches	Percent	Percent	Percent	Percent	Percent
.2	100				
.4	100				
.6	92	8			
.8	68	32			
1.0	50	50			
1.2	37	63			
1.4	28	71	1		
1.6	21	75	4		
1.8	18	73	9		
2.0	15	69	16		
2.5	10	56	32	2	
3.0	8	42	41	9	
3.5	7	31	43	17	2
4.0	6	24	39	25	6

1/ See page 5 in "Methods of Predicting Growth of Forest Stands," Economic Note No. 9, April 1938, Lake States Forest Experiment Station.

A stand table (table 10) can be prepared for unmerchantable trees 10 years hence by applying the percentage of trees expected to move into higher d.b.h. classes (table 9) to the present distribution.

Table 10.--Number of unmerchantable red oak trees that become merchantable in 10 years, fully stocked 60-year-old stand, medium site

D.b.h. class	Average diameter increment	Trees per acre	Trees moving into merchantable 12-inch class	Gross volume acquired
Inches	Inches	Number	Number	Board feet Int'l. 1/4-inch
6	.67	32	None	
8	.92	66	None	
10	1.17	62	38	2,584

In the Lake States none of the 6- and 8-inch trees in a 60-year-old red oak stand, with the diameter increments of .67 and .92 inches respectively, will graduate into the 12-inch class in 10 years' time. Of the trees in the 10-inch class which have an average increment of 1.17 inches, 62 percent (38 trees) will graduate into the 12-inch group (table 9). This will add a gross volume growth of 2,584 board feet to the merchantable stand, or an annual growth of 258 board feet.

Calculation of Net Growth

To obtain net growth, the gross growth must be reduced to allow for mortality and cull losses. In the previous computation it was assumed that no mortality would take place and that all trees were essentially sound. The growth was allowed on those trees which were expected to die or become more defective during the period. Net growth can be computed from the following formula:

$$g = G - M - (V_2 c_2 - V_1 c_1) \text{ where}$$

G = gross growth

M = mortality

V_1 = present gross volume

V_2 = future gross volume

c_1 = cull percent of present gross volume

c_2 = cull percent of future gross volume.

The computation of mortality for merchantable trees is made from the formula:

$$M = m\,(V_1 + G) \text{ where}$$

m = mortality in percent of gross volume (V_1).

For example, if the present gross volume on merchantable trees is 4,362 board feet, the annual gross growth is 211 board feet (table 8), and the mortality data indicate that the annual mortality is 0.75 percent of the gross volume, the mortality reduction is .0075 (4,362 + 211) or 34 board feet. Again, if 38 of the 62 trees in the 10-inch class move forward in 10 years into the 12-inch class, and mortality studies show that 2 of these trees will die in 10 years' time, mortality loss is 2 x 68 (the International board-foot volume of a 12-inch tree) or 136 board feet -- approximately 14 board feet annually. The mortality loss for the stand, therefore, is 34 + 14 = 48 board feet per annum.

In some cases, it may be more practical to allow for mortality losses before applying the growth percent. The number of trees per acre (column 2, table 8, and column 3, table 10) can be reduced for dead trees and only those expected to survive used in the computation of growth. The computed growth would then be that on the surviving trees and no mortality volume allowances would need to be made later.

As a rule the average cull percent of stands increases with age. At age 60, the cull of red oak is approximately 10 percent of the gross board-foot volume; at age 100, it is on the average 16 percent of gross volume; and at age 160 the cull percent rises to about 30 (table 11).

Table 11.--Average cull percent of merchantable red oak trees by age of stand

Stand age	Cull percent	Stand age	Cull percent
Years		Years	
40	8.5	100	16.0
50	9.1	110	18.0
60	10.0	120	20.2
70	11.2	130	22.6
80	12.6	140	25.1
90	14.2	150	27.8
		160	30.6

If a 60-year-old stand of red oak has defect which is 10 percent of the gross volume (including ingrowth), it will have an expected future cull for the next year which is 0.12 percent more or 10.12 percent of the future gross volume. As mentioned above, the deduction for cull losses is

$$V_2c_2 - V_1c_1$$

For the example given previously, cull losses for both the merchantable trees and ingrowth are

$[4,362 + (211 + 258) - (34 + 14)] \times .1012 - 4,362 \times 0.10$, or 48 board annually.

Net growth is obtained by substracting deductions for mortality and cull losses from gross growth. In the red oak example this amounts to

$$(211 + 258) - (34 + 14) - 48, \text{ or } 373 \text{ board feet.}$$

ACCURACY OF THE GROWTH PREDICTION

The accuracy of the proposed method for predicting current growth in forest stands is dependent upon the intensity of sampling used for obtaining (1) stock tables, (2) diameter and height increment, and (3) reduction for mortality and cull losses. Any errors introduced in these separate phases of the work will affect the accuracy of the estimated final net growth.

The accuracy of the diameter increment can be controlled easily because it depends on the number of borings obtained and the relative variability of growth over the area. The procedure for evaluating sampling errors from all the different sources is rather intricate and will not be presented in this paper. However, a conservative rule-of-thumb for estimating the number of borings needed and the accuracy of gross volume growth is given in the appendix. This rule-of-thumb will be particularly useful for planning the size of sample needed.

APPENDIX

Correlation of Diameter Increment with Decadal Growth Index and Age

This method was used in analyzing the growth of 2,200 red oak trees in southwestern Wisconsin. By placing the data for each sample tree on a small card, the work of sorting, applying curve readings, and computing was greatly facilitated. A detailed description of the process follows:

1. The decadal growth of each tree was computed, the trees were sorted by decadal-growth groups, and the average growth during the past 10 years was computed for each group. These averages were plotted over decadal growth, and a trend line drawn (figure 1). The readings for growth during the past 10 years were lower than decadal growth.

2. For each sample tree a first estimate of the last 10 years' growth was read off the curve, and its residual (the difference between the reading and the actual past 10 years' growth) was computed.

3. A calculation of the standard deviation of the actual increment in each decadal-growth group showed that more variation existed in terms of inches among the faster growing trees than among slower growing ones. Each group, however, had about the same coefficient of variation, that is, the same variation in terms of percentage of its own mean. Accordingly, each residual was converted into a percent of the first estimate, e.g., the residual for a tree with a past 10 years' increment of 2.5 inches and a first estimate of 2 inches was computed as a plus 25 percent, and the residual for a tree with a past 10 years' increment of 1.25 inches and a first estimate of 1 inch was also computed as a plus 25 percent.

4. The trees were then sorted into age groups, and the average percent deviation for each age group was computed. These averages were then plotted over age and a curve drawn (figure 2), on which the deviations for all trees less than 45 years of age were positive, whereas the deviations for trees over 45 years of age were negative. At 55 years of age and over, the curve levels off at the minus 5 percent line. The first estimate for all trees in this age group, therefore, should be lowered 5 percent.

NOTE: Since in most cases, the curve of the average growth during the past 10 years over the decadal growth (step 1) is a straight line, the above procedure can be simplified greatly by the following short cut: Instead of computing residuals for each individual tree and averaging their deviation percents, the trees can first be sorted by age classes, and the first estimate made from figure 1 for the average decadal growth computed for each age group. The difference between this residual and the average actual growth for the group (step 2) can then be converted to a percentage as indicated in step 3. This short cut will greatly reduce the amount of computation since it eliminates the necessity of handling the data by individual trees.

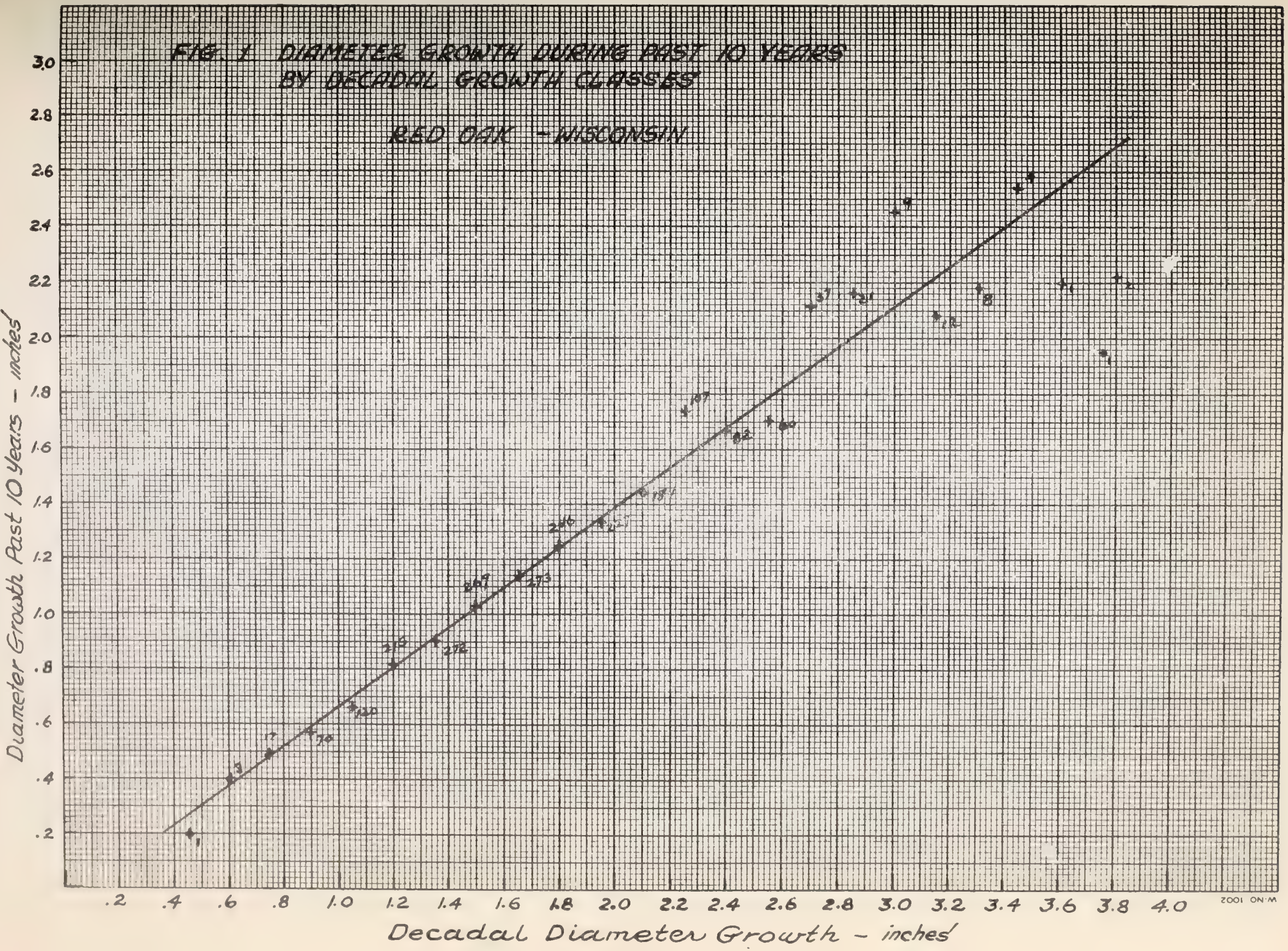

FIG. 1 DIAMETER GROWTH DURING PAST 10 YEARS BY DECADAL GROWTH CLASSES

RED OAK - WISCONSIN

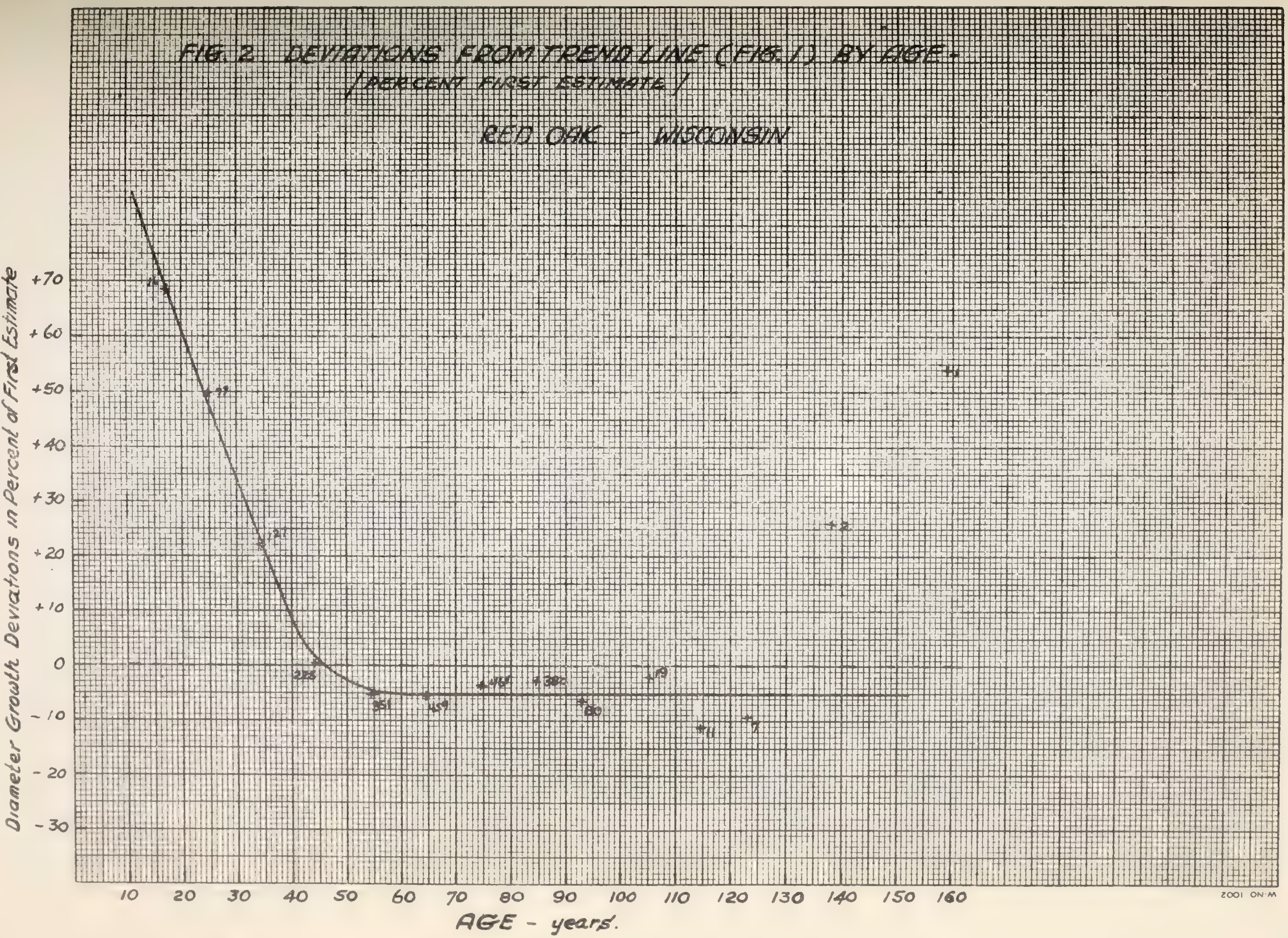

FIG. 2 DEVIATIONS FROM TREND LINE (FIG. 1) BY AGE -
(PERCENT FIRST ESTIMATE)

RED OAK - WISCONSIN

5. To apply these curves, first a table showing decadal growth for each diameter class and age group was computed. This table need be set up only once and can thereafter be used for any species, e.g., any tree 10 inches in diameter and 50 years old must always have a decadal growth of 2 inches.

6. For each of the decadal growth figures computed in step 5, the first estimate of the past 10 years' growth was read from the decadal growth curve (figure 1).

7. As bark growth was not included in the calculation of increment, it was added at this point. (See table 12 for bark factors of the species in the Lake States.) The correction for age read from figure 2 and for bark growth was made in one step by multiplying the two factors together and applying the result to the first estimates made in step 6.

Table 12.--Bark factors 1/ for tree species in the Lake States

Species	Bark factor	Species	Bark factor
Ash, black	1.087	Hickories	1.087
Ash, green	1.117	Hophornbeam, eastern 2/	1.100
Ash, red	1.117	Maple, red	1.076
Ash, white	1.117	Maple, silver	1.076
Aspen, quaking	1.096	Maple, sugar	1.096
Basswood, American	1.104	Oaks	1.122
Beech, American	1.046	Pine, jack	1.107
Birch, black	1.099	Pine, eastern white	1.117
Birch, paper	1.075	Pine, red	1.112
Birch, yellow	1.087	Poplar, balsam 3/	1.096
Cherry, black	1.087	Poplar, yellow	1.104
Cottonwood, eastern	1.096	Spruce, black	1.065
Elms	1.100	Spruce, white	1.065
Fir, balsam	1.070	Tamarack	1.056
Hemlock, eastern	1.124	Walnut, black	1.087
		White-cedar, northern	1.099
		Willow, black	1.096

1/ Multipliers for converting diameter increment of wood to total diameter increment. (Diameter increment of wood x bark factor = diameter increment of both wood and bark.)
2/ Commonly known in the Lake States as ironwood.
3/ Commonly known in the Lake States as balm of Gilead.
Source: "Methods of Predicting Growth of Forest Stands," Economic Note No. 9, April 1938, Lake States Forest Experiment Station.

8. Since this growth had accrued on trees during the past 10 years, the diameter 10 years ago was obtained by substracting the increment of wood and bark (step 7) from the present diameter. This increment was then curved over the past diameter for each age class, and each trend line on the graph was labeled according to the age of the trees 10 years ago. 4/

9. From these curves a table of future diameter increment by d.b.h. class and age group was made (table 1).

Rule-of-Thumb for Estimating the Number of Borings Needed and the Accuracy of Gross Volume Growth

A rule-of-thumb for estimating the standard error of sampling gross volume growth for a given size class of timber is shown below:

$$E^2 = 2.25 \text{ Sum } \frac{P^2}{n} \quad \text{where}$$

E = the standard error of total volume growth, in percent (including all diameter classes)

P = the actual or estimated percent of total volume in any diameter class

n = the required number of borings in any diameter class, and

2.25 = a constant

This rule-of-thumb is recommended only for a rough estimation of sampling error. The method incorporates the errors of both diameter and height increments based on estimated average variability, and assumes that those error estimates are equally accurate.

The rule is especially useful, however, for estimating the number of borings needed and their allocation by diameter class when planning growth studies. It requires only knowledge of the approximate distribution of volume by different diameter classes. The number of borings in any diameter class can be estimated in the following manner:

1. Decide on the percent error desired for the stand.

2. Divide 225 by the percent error squared.

3. Multiply the percent of volume in each diameter class by the figure obtained in step 2. This is the number of borings needed in each class.

4/ In any future growth studies by this method, it is recommended that all sample-tree calculations be based on decadal growth and age as of 10 years ago. In applying figure 2 to stand tables, however, estimated growth 10 years hence should be based on present age.

The following example demonstrates the use of the rule-of-thumb for determining the number of borings needed in a hardwood stand, large saw-timber size, where approximately a 10 percent standard error of total volume growth is desired.

Table 13.--Estimate of number of borings needed in each diameter class to obtain a standard error of volume growth of approximately 10 percent 1/

(Large Saw-Timber Hardwood Stand.)

D.b.h.	P = percent of total volume	n = number of borings (2.25 P)
Inches		
12	5	11
14	10	22
16	19	43
18	17	38
20	18	40
22	14	32
24	9	20
26	8	18
Total	100	224

1/ Using the rule-of-thumb: Percent error (10) squared = 100; 225 divided by 100 = 2.25.

For a stand with the volume distributed as shown above, a total of 224 borings would be required to obtain a standard error of total volume growth of 10 percent. The error of volume growth for each diameter class would, of course, be considerably larger.

If the above stand comprises a relatively small area and it is not practical to obtain as many as 224 borings, it will be necessary to make several trials with higher standard errors assumed.